

## THE ROLE OF THE CONCEPT OF ALGEBRAIC CLOSED FIELD THEORY IN THE DEVELOPMENT OF LITERACY OF STUDENTS OF MATHEMATICAL SPECIALITIES

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The algebraic closure of the field of real numbers is the field of complex numbers. Its algebraic closure is denoted by the main theorem of algebra. The algebraic closure of the field of rational numbers is the field of algebraic numbers. The field of arithmetic numbers is algebraically closed. Algebraically closed fields will be a field- $K$  if any nonzero polynomial of  $K$  has one root of a smaller one. Unique to any field up to isomorphism, it is algebraically closed. If  $K$  is algebraically closed, then any irregular polynomial coefficients of  $K$  are partitions of  $K$ . For example, the field of real numbers is not algebraically closed because the polynomial  $X^2 + 1$  has no real root; on the contrary, the field of complex numbers is algebraically closed: this is a fundamental theorem of algebra, also called the D Lambert-Gauss theorem [1].

Complete theories are important to arrive at the notion of algebraic closed fields. So, with respect to the node of our title, while the field-addition ( $+$ ) and multiplication ( $\cdot$ ) operations represent any set  $k$  satisfying the given field axioms, an algebraically closed field is a field  $K$  that has one root of any non-zero polynomial of  $K$ .

Every field  $K$  has an algebraic closure which is the smallest algebraic closed field of  $K$  which is a subfield. The algebraic closure of this field is a unique feature of  $K$ -isomorphisms. In particular, the field of complex numbers is an algebraic closure of the field of real numbers, and the field of algebraic numbers is an algebraic closure of the field of rational numbers. A finite field  $K$  cannot be algebraically closed. Indeed, if we consider the product  $P(X) = \prod_{k \in K} (x - k)$ . then  $p + 1$  is a non-constant polynomial with no root in  $K$ . The first-order theory of algebraic closed fields makes use of the epimination of quantifiers. Thus, the theory of algebraic closed fields with fixed characteristic is complete, and a first-order statement is true for algebraic closed fields with zero characteristic only if it is true for fields with sufficiently large characteristic.

To fully convey the topic, we will need language, structure, term, formula, model and other concepts such as. Of great importance is the theory of convergence [2].

By considering the theory and the complete theory, we give a broad overview of the topics of groups, abelian groups and fields. Thus, we can fully extend the definition, the notion of the theory of algebraic closed fields. First-order logic (predicate calculus) is formal calculus that admits statements about variables, fixed functions, and predicates. It extends the logic of statements. In turn, it is a special case of higher-order logic. A first-order logic language is built around a signature consisting of a set of function symbols  $\{F\}$  and predicate symbols  $\{P\}$ . Each function and predicate symbol depends on the location of  $\cdot$ , that is, the number of possible arguments. First-

order logic allows Strong judgements about the truth and falsity of statements and their relationship, in particular, about the logical consequences of one statement over another or, for example, about their equivalence.

Definition . A field is any set  $k$  (axioms of fields) satisfying the following conditions, where the operations of addition ( $+$ ) and multiplication ( $\cdot$ ) are given:

For any  $a, b \in k$ , the equality  $a + b = b + a$  holds (commutativity of addition);

For any  $a, b, c \in k$ ,  $(a + b) + c = a + (b + c)$  (associativity of addition);

For any  $A \in k$ , we have an element  $0$  in  $k$  such that  $a + 0 = a$  (existence of zero);

For any  $A \in k$ , we have  $b \in k$  such that  $a + b = 0$  (existence of an opposite element: such an element  $b$  is called the opposite of  $A$  and is denoted by " $-a$ ");

For any  $a, b \in k$ , the equality  $a - b = b - a$  (multiplication permutation) is true;

For any  $a, b, c \in k$ , we have  $(a - b) - c = a - (b - c)$  (associativity of multiplication);

For any  $A \in k$ , there exists an element " $1$ " that is not zero in  $k$  such that  $A \cdot 1 = a$  (existence of a unit);

For any non-zero  $A \in k$ , there is a  $b \in k$  such that  $A \cdot b = 1$  (existence of an inverse), such an element  $b$  is called the inverse element of  $A$  and is denoted as  $1/a$  or  $a^{-1}$ ;

For any  $a, b, c \in k$ ,  $A \cdot (b + c) = a \cdot b + a \cdot c$  (the distribution property of multiplication over addition) holds.

Definition . for any field elements  $a, b \in F$ ,

the equation  $a + x = b$  has a single solution  $x = b + (-a)$ ,

which is denoted  $b - a$  and is called the difference of the elements  $b$  and  $A$ . Thus, the get operation is defined in the field.

Definition . for any field elements  $A, b \in F$ ,  $a \neq 0$ ,

the equation  $a \cdot x = b$  has a unique singular solution

$x = b \cdot a^{-1}$ , which is denoted  $b/a$  and is called the separation of elements  $b$  and  $A$ .

Let  $L = \{+, -, \cdot, 0, 1\}$  be the language of ring theory. Consider the following  $L$ -theories: the ACF-algebraic theory of closed fields, the ACF<sub>p</sub>-algebraic theory of closed fields of description  $P$ , where  $p$  is a prime number or  $0$ . We define ACF<sub>ti</sub> a<sub>1</sub>. . . together with the axioms  $x a_n x x$

$$(x^n + a_1 x^{(n-1)} + \dots + a_n = 0)$$

using the known field axioms for all natural numbers  $n > 0$ .

Then the ACF<sub>p</sub> theory for a prime number  $p$  via the ACF axioms

$\forall x ((x + \dots)^p = x^p + \dots)$  is defined by the axiom  $(x + \dots)^p = x^p + \dots$ .

The ACF<sub>0</sub> theory is represented by the ACF axioms, and the axioms  $p$  beryledi  $x$

$((x(x + \dots + x))^p = 0)$ ,

for all prime numbers  $p$ .

The algebraic theory of closed fields of constant characteristic is also complete; the proof of this fact is similar to the proof of statement 2, the measure of which is replaced by the degree of transcendence.

Conclusion . The theory ACF<sub>p</sub> is  $\alpha$ -categorical for all uncountable  $\alpha$ -cardinals.

Proof. It is known that for every field extension one can choose a transcendental basis whose semisimplicity is uniquely determined; moreover, two algebraically closed fields are isomorphic if they have the same characteristic and degree of transcendence (over a simple subfield). If this degree of transcendence is equal to  $\alpha$ , it is easy to see that the field has power  $\alpha + 0$ . Hence, the degree of transcendence of an algebraically closed force field  $\alpha > 0$  is uniquely defined; thus 1, 2,....., Isomorphic fields of transcendental degree  $0$  are numerical [3].

Conclusion. In this paper, many references, concepts are given to reveal the theory of algebraic closed fields, covering many theorems starting from first-order series logic. In addition, the notions of term, formula, atomic formula, language are widely discussed.

The notion of models and theories using structures and languages is given. Many theorems can be found in this work, each of which is, shall we say, important in the theory of closed algebraic fields. For example, the convergence theorem, which states that a theory  $T$  is a model only if there exists a

model of every last subset of theory T, Gödel's completeness theorem, the Levenheim-Skolem theorem, which further strengthens the convergence theorem and proves that theory T has a degree model exactly equal to  $\alpha$ , and other similar theorems.

Consider the following statement for the above statement:

Report № 1.  $f(x) = x^4 - x^3 - 3x^2 + 2x + 2 \in \mathbb{Q}[x]$  it is necessary to factorize a polynomial without rational roots or to find out its irreducibility in the field

The solution. We will solve this problem with the beautiful Kronecker. In the field  $\mathbb{Q}$ , the reduced integer coefficient is divided into the product of factors whose coefficients of the polynomial are integers. Since the polynomial  $f(x)$  has no rational roots, it does not have linear factors, and if it is linear in the field  $\mathbb{Q}$ , it divides into two quadratic factors:

$$f(x) = (x^2 + px + q)(x^2 + p_1x + q_1) = g(x) \cdot q(x) \in \mathbb{Z}[x]$$

Any integer  $x = m$  actually  $f(m) = g(m) \cdot q(m) \in \mathbb{Z}$ ,  $f(m) \div g(m)$  that will happen

$g(x) = x^2 + px + q$  we use to find the polynomial.

$f(1) = 1$ ,  $f(-1) = -1$ , since  $g(1)$  i  $g(-1)$  the following combinations are suitable for:

- 1)  $g(1) = 1$ ,  $g(-1) = 1$ ;                      2)  $g(1) = -1$ ,  $g(-1) = 1$ ;
- 3)  $g(1) = 1$ ,  $g(-1) = -1$ ;                    4)  $g(1) = -1$ ,  $g(-1) = -1$ ;

$$1) \begin{cases} g(1) = 1 + p + q = 1 \\ g(-1) = 1 - p + q = 1 \end{cases} \Leftrightarrow \begin{cases} p = 0 \\ q = 0 \end{cases} \Leftrightarrow g(x) = x^2$$

Then  $f(x)$  the polynomial is  $g(x)$ -ne is not divided, ie  $f(x) \neq x^2 \cdot q(x)$

$$2) \begin{cases} g(1) = 1 + p + q = -1 \\ g(-1) = 1 - p + q = 1 \end{cases} \Leftrightarrow \begin{cases} p = -1 \\ q = 1 \end{cases} \Leftrightarrow g(x) = x^2 - x - 1,$$

$f(x)$  the polynomial is  $g(x)$ -ne is divided:

$$- \frac{x^4 - x^3 - 3x^2 + 2x + 2}{x^4 - x^3 - x^2} \quad \frac{x^2 - x - 1}{x^2 - 2}$$

$$\begin{array}{r} \hline -2x^2 + 2x + 2 \\ \hline -2x^2 + 2x + 2 \\ \hline 0 \\ r(x) = 0 \Rightarrow f(x) : g(x) \\ \text{So, } f(x) = (x^2 - x - 1)(x^2 - 2) \in Q[x] \end{array}$$

Warning. If the polynomial is not divisible by considering all the  $f(x)$  the polynomial is  $g(x)$

-ne if it is indivisible, then we say it is  $f(x)$  the polynomial is  $Q$  irreducible in the polynomial field.

#### List of references used

1. Manturov O.V., Matveev N.K. Higher mathematics course. Linear algebra. Analytical geometry. Differential calculation of a function of one variable. – M.: Vysshaya Shkola, 2019.
2. Smagulov E.Zh. Ways of development of mathematical thinking of students in the process of learning mathematics // Herald of KazNU. Al-Farabi. Series pedagogical sciences. – Almaty, 2019. No. 3(22). P. 89-94
3. [https://ust.kz/word/matematikalyq\\_zertteylerdegi\\_matematikalyq\\_indykciya\\_adisi-174512.html](https://ust.kz/word/matematikalyq_zertteylerdegi_matematikalyq_indykciya_adisi-174512.html)